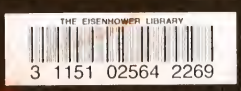
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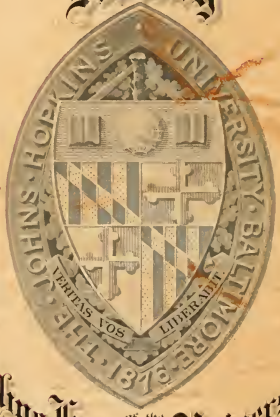

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ROWLAND'S NEW METHOD FOR MEASURING MAGNETIC ATTRACTION AND ENERGY LOSSES DUE TO HYSTERESIS AND FOUCAULT CURRENTS, AND DETECTION OF SHORT CIRCUITS IN COILS.

DISSERTATION

Submitted to the

BOARD OF UNIVERSITY STUDIES

of the

JOHNS HOPKINS UNIVERSITY,

for the

Degree of Doctor of Philosophy,

by

Louis Maxwell Potts.

May, 1900.

93522

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ROWLAND'S NEW METHOD FOR MEASURING ELECTRIC ABSORPTION AND ENERGY LOSSES DUE TO HYSTERESIS AND FOUCAULT CURRENTS, AND DETECTION OF SHORT CIRCUITS IN COILS.

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The following investigation has a twofold object. One is the testing of the method for the measurement of electric absorption, described in the American Journal of Science, July, 1899, and also of a similar method devised by Professor Rowland for measuring energy losses in iron due to hysteresis and Foucault currents, and for the detection of short circuits in coils.

Historical:- It has long been known that after a Leyden jar has been charged and then discharged, it will show another charge after standing a short time. If this is discharged, after a short time the jar may show another charge. This may be repeated indefinitely. These "after-charges" are known as residual charges and are due to the phenomena now called electric absorption. Faraday[1] made some experiments on this phenomenon in Leyden jars, and seems to have attributed it to a conduction of the charge into the interior of the dielectric, and after discharge creeping back again to the coatings and manifesting itself as the residual charge. Kohlrausch[2] was the first to make any elaborate investigation of the subject. He charged the condenser and then measured the potential at certain intervals with an electrometer. In this way he obtained the relation between the potential and time. He advanced the idea that the phenomenon was due to an electric polarity of the particles of the dielectric, produced by the electric force between the plates of the condenser. Rowland and Nichols[3] have shown that certain crystals show no electric absorption along lines of homogeneity, and do along other lines.

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1) Faraday - Experimental Researches.

2) Pogg. Annalen, Vol. 91, p. 56-82, 179-213.

3) Phil. Mag. p. 411, 1881.

E. Hertz[1] has shown that pure liquids possessed no electric absorption, while impure ones do.[2] The great sensitiveness of this phenomena to change of temperature has been noted. The energy loss in condensers due to it has also been studied.[3]

THEORY OF ELECTRIC ABSORPTION.

The theory of electric absorption has been developed by Clausius[4], Riemann[5], Maxwell[6] and Rowland[7]. The following development is that of Maxwell applied by Professor Rowland to the case of a dielectric acted upon by an E.M.F. varying harmonically.

A dielectric such as paraffin paper is made of a substance of certain dielectric capacity and specific resistance, having imbedded in it particles of a different dielectric capacity and different specific resistance. Now we can very closely approximate to this case by considering a plane plate condenser, in which the dielectric is made up of a number of layers of different substances. An ordinary condenser is merely a great number of very small condensers like this, joined in multiple.

The theory of electric absorption as extended by Professor Rowland shows that a condenser possessing dielectric absorption should act as a capacity in series with a certain resistance. The value of each depends on the fre-

1) Wied. Annalen, p. 231, 1883.
2) Phil. Trans. p. 599, 157. Proc. Roy. Soc. p. 438, 1879.
3) Physical Review, 1893, p. 73.
4) Theorie Mechanique de la Chaleur Deuxieme partie
5) Riemann's Mathematical Works, p. 80
6) Elec. and Mag. Vol. 1, p. 393.
7) Am. Journ. Science, (3rd series) p. 1[illegible]

riod of the current. If b_0, b_1, etc. are constants and T the period of the current, the resistance is of the form

$$R = b_0 T^2 - b_1 T^3 + b_2 T^4 \text{ etc.}$$

and if a_0 a_1 etc. are constants the capacity is of the form

$$\frac{1}{C} = a_0 - a_1 T + a_2 T^2 \text{, etc.}$$

GENERAL THEORY.

The arrangement is essentially a Wheatstone bridge, in which the fixed coils of an electro-dynamometer were placed in one arm of the bridge and the hanging coil in the cross-connection, in place of the galvanometer in the direct current method of use. The adjustment of the bridge then used depends upon the fact that there will be no deflection of the electro-dynamometer if the phase difference of the current in the fixed coils and that in the hanging coil is 90°.

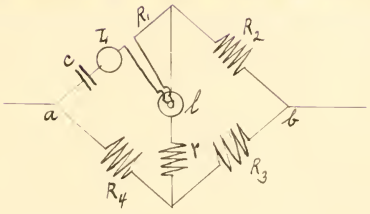


Figure 1

Fig. 1 is the arrangement used. Let R_1, R_2, R_3, and R_4 be the resistances of the different arms and r that of the cross-connection. Let c_1 be the current in the arm a of the bridge, and c_2 in the cross connection.

If we apply at a and b a direct electromotive force E we shall have the following expressions for the currents.

$$c_2' = \mathcal{E} \frac{R_2 R_4 - R_3 R_1}{\Delta} \qquad (1)$$

$$c_1' = \mathcal{E} \frac{R_4(R_2+R_3) + r(R_3+R_2)}{\Delta} \qquad (2)$$

where

$$\Delta = r(R_3+R_4)(R_1+R_2) + R_1R_2(R_3+R_4) + R_3R_4(R_1+R_2)$$

If in place of using direct current we apply to the terminals a b a simple alternating E. M. F. $\mathcal{E}e^{ibt}$ we shall get the corresponding quantities by the following substitutions.

$$c_1' = c_1 e^{i(bt+\varphi_1)} \qquad c_3' = c_3 e^{i(bt+\varphi_3)}$$

and if we place in the arm 1 a coil of self-induction L and a capacity c we must substitute for R_1, R_1', where

$$R_1' = R_1 + i\,b\,L - \frac{i}{b\,c}$$

and if l is the self-induction of the hanging coils of the electro-dynamometer we must replace r by r', where

$$r' = r + i\,b\,l.$$

Making these substitutions and dividing (1) and (2) we have

$$\frac{c_3}{c_1} e^{i(\varphi_3-\varphi_1)} = \frac{R_2R_4 - R_1R_3 - i\left(b\,L - \frac{1}{b\,c}\right)R_3}{R_4(R_2+R_3) + r'(R_3+R_4) + i\,b\,l(R_3+R_4)} \qquad (3)$$

Now taking only the real part of the above quantities we have

$$\frac{c_3}{c_1}\cos\varphi = \frac{(R_2R_4 - R_1R_3)\left\{R_4(R_2+R_3) + r(R_3+R_4)\right\} + \left(\frac{L}{c} - b^2 l L\right)R_3(R_3+R_4)}{\left\{R_4(R_2+R_3) + r(R_3+R_4)\right\}^2 + b^2 l^2 (R_3+R_4)^2} \qquad (4)$$

If $\varphi = 90°$ $\cos\varphi = 0$ or

$$(R_2R_4 - R_1R_3)\{R_4(R_2 + R_3) + r(R_3 + R_4) + (\frac{\ell}{c} - L^2\ell L)R_3(R_3 + R_4) = 0$$

or

$$R_1 = \frac{R_2R_4}{R_3} + (\frac{\ell}{c} - L^2\ell L) \frac{R_3 + R_4}{R_4(R_2 + R_3) + r(R_3 + R_4)} \qquad (3)$$

This then is the condition satisfied when there is no deflection of the dynamometer.

The first term of the above is the same as the expression for R_1 when the adjustment is conditioned by no deflection of a galvanometer in the cross-connection and a direct current is used. The second term is a small correction term, always very small, at most one per cent, and often entirely negligible. In a circuit carrying an alternating current, the effective values of R etc. are not usually the same as their actual Ohmic values, but are larger. They include that part of the impedance against which work must be done to maintain the current. Let R_2 R_3 and R_4 be as nearly as possible pure ohmic resistances, i.e. let their actual values be equal to their effective ones. And let the arm 1 contain iron, a condenser possessing electric absorption, or any piece of apparatus using energy, which is not expended in heating the conductors of that arm. The value of R_1 calculated by the above formula will be the effective resistance and this, less the

actual value as measured by a direct current, will be the increased resistance due to the hysteresis of the iron, the energy-loss by electric absorption, etc.

In the measurement of certain quantities (e.g. hysteresis loss in iron) by this method, it is necessary to insert in arm 1 of the bridge a large coil. Such a coil acts not as a pure self-induction, but on account of the numerous coils of the wire so close to one another, as a self-induction in multiple with a capacity. On this account formula (5) is not strictly accurate but should include a term involving the capacity of the coil. Let the capacity which in parallel with the self-induction L will have the same effect as the given coil be k. And let R' be the resistance of the coil and R'' the resistance of the remainder of arm 1. Substitute in (5) in place of

$$R_1 + i\,b\,L \quad \text{by} \quad \frac{i}{b\,c}$$

$$R_1'' + \frac{R_1' + i\,b\,L}{(1 - b^2 k L) + i\,L\,K\,R_1'}$$

And also put

$$R_4(R_3 + R_2) + r\,(R_3 + R_4) = A$$

Then

$$\frac{R_1}{R_4} = \frac{(R_3 R_1'' - R_2 R_4)(1 - b^2 k L) + R_1' R_3 + i\{b\,K\,R_1' R_1'' R_3 + b L R_3 L K R_1' R_2 R_4}{A(1 - b^2 k L) - b^2 k L R_1' (R_3 - R_4) + i\{A L k R_1' + (1 - b^2 k L) L / (R_3 + R_4)\}}$$

As before, the condition for no deflection is that the real part of (E) vanishes.

or

$$\left[(R_2R_4 - R_3R_1'')(1-b^2kL) - R_1' R_3\right]\left[A(1-b^2kL) - b^2k\ell R_1'(R_3 + R_4)\right]$$
$$= \left[Abk R_1' + (1-b^2cL)\, b\ell (R_3 + R_4)\right]\left[bLR_3 - bkR_1'(R_2R_4 - R_1''R_3)\right]$$

Expanding this it becomes,

$$A(1-b^2kL)^2(R_2R_4 - R_3R_1'') - A R_1'R_3(1-b^2kL)$$
$$+b^2k\ell R_1'(R_3 + R_4)R_3 - (R_2R_4 - R_3R_1'')(1-b^2kL)\, b^2k\ell R_1'$$
$$(R_3 + R_4) = A\, b^2k\, L\, R_1'R_3 - A\, b^2k^2R_1'^2 (R_2R_4 - R_1''R_3)$$
$$(1 - b^2kL)\, (R_3 + R_4)R_3 b^2 L - (1-b^2k\, L)(R_3 + R_4)\, (R_2R_4 - R_1''R_3)$$
$$b^2k\ell R_1'$$

Now since k is in all cases small and ℓ is also small, the terms above which involve k^2 and $k\ell$ may be dropped, whence we get,

$$R_1' = \frac{A' (1-b^2kL)^2 (R_2R_4 - R_3R_1'') - b^2\ell L\, (R_3+R_4)R_3(1-b^2kL)}{A\, R}$$

$$= 1 - b^2cL)\, \frac{R_2R_4 - R_3R_1''}{R_3} - b^2\ell L\, \frac{R_3 + R_4}{A}(1-b^2k\, L)$$

Now since the last term is very small and $1-b^2k\, L$ is nearly 1, it may be dropped; and we have for the final formula.

$$R_1' = \frac{R_2R_4 - R_3R_1''}{R_3} + \left(\frac{\ell}{c} - b^2\ell L\right) \frac{R_3 + R_4}{R_4(R_3+R_2) + R_3(R_3+R_4)}$$

$$- 2\, b^2k\, L\, \frac{R_2R_4 - R_3R_1''}{R_3} \quad \ldots \quad (7)$$

In the measurement of dielectric absorption and hysteresis loss it is necessary to know the period of the current used. In this investigation a speed counter and chronograph were used. The speed counter was placed at the end of the dynamo shaft and directly connected to it. A contact was so arranged that for every 100 revolutions of the dynamo armature a circuit was closed and a record made on a chronograph sheet. On a table beside the electro-dynamometer was a key, which also could be used to make a record on the chronograph sheet. As soon as the bridge had been balanced, this key was pressed. And by the measurement of this sheet the period of the current at the time of the observations was quite accurately determined. The error from this source was usually not more than one part in 1000, never more than 1 in 100.

After each adjustment, the resistance of each arm was determined by the use of a Post-office Box". (When the current through any part of the apparatus was not large, it was not necessary to measure its resistance after each adjustment.) The resistance of that part which was affected the most by heating was measured first, and in this way the actual value at the time of adjustment was quite closely determined. In cases where the heating was large, the error from this cause might reach several parts in 1000 in R' . and consequently a considerably greater amount in the value

of the electric absorption resistance.

Errors due to induction and electrostatic action of the different portions of the apparatus were carefully guarded against by the arrangement. And induction was tested for by reversal of the relative direction of the current in different portions of the apparatus. Usually no effect was noticed, or at most it was very small. The errors introduced by the self induction of the electro-dynamometer coils and also that caused by the electrostatic action of the turns of a large coil on one another, were determined and corrected for when sufficiently large in amount.

APPARATUS.

Electro-dynamometer[1] The self induction of the fixed coils was .0165 henry, and of the hanging coil .0007 henry.

Dynamos:- The current used in this investigation was furnished by three dynamos. The Westinghouse alternator in power-house of the University furnished a current of period .0075, i.e. 133 complete periods per sec. This was used for only a few observations. In most of the work two small dynamos constructed in the University work-shop were used. Both were directly connected to some small electric motors. Both had armatures of the pancake type. The one had four coils on the armature and four poles, and thus produced a current of two complete periods for each revolution of the armature. The other was larger but of similar construction having six coils on the armature and six poles, and gave three complete periods for one revolution. If the load on the dynamo were not changed, these dynamos would run at a very nearly constant speed. With the second dynamo the number of complete periods per second could be varied from 6 to 70. However, at the lower speed the electro-dynamometer was difficult to balance, since the hanging coil would vibrate with the current and blur the image of the

1) See Am. Journ. Science, July, 1906, p. 35.

scale. The voltage furnished could be controlled very well by changing the strength of the field. In any one series of observations the same dynamo was used, as the results using different dynamos would not be comparable, on account of the different harmonics introduced. The small dynamos which were almost exclusively used gave, however, a very good sine curve.

Resistances:- The high resistances and those which were required to carry very small currents were made of fine german silver wire wound on thin sheets of fibre. The self induction and electrostatic action of these was practically zero. The lower resistances, and those required to carry larger currents were made of a special resistance wire, which had a very slight negative temperature coefficient, and would bear considerable heating with a very small change of resistance. This wire was wound on slates. Each slate contained sufficient number 30 wire to have nearly 2000 ohms resistance. These were conveniently subdivided for adjustment. For the final adjustment an ordinary resistance box was used, but never more than fifty ohms were used in this box, and then the total resistance of that arm was at least 1000 ohms.

Self inductance:- Two coils were used.

A - External diameter 3546 cm.; internal diameter 23.8 cm. 3700 turns #20 B. & S. Self inductance 5.30 henrys. Re-

sistance 108 ohms.

C - Same dimensions as A except depth. Self-inductance 1.30 henrys. 1747 turns #32 B. & S. single cotton covered copper wire. Resistance 78 ohms.

Condensers:- 2 and 3. Paper condensers made by Marshall of 2 and 3 micro-farads capacity.

Willyoung - 8 microfarad wax condenser made by Willyoung & Company, and divided into sections of one microfarad each.

Mica Condensers - 1/3 M. F. standard condenser made by Elliot Brothers.

1/3 M. F. Standard condenser made by Troy Electric Company.

The bridge - The bridge was set up permanently on a table; all connections were soldered; and the wires used in connecting different parts of the bridge and instruments were #25 cotton covered copper wire. All wires were made as short as possible and no wires were twisted.

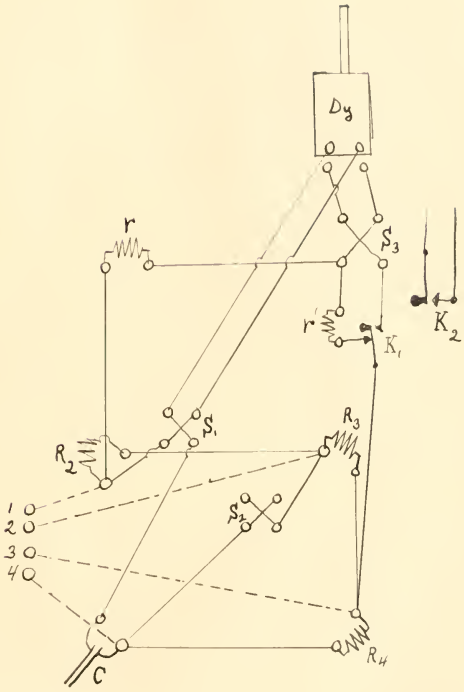


Figure 2.

Investigation:- In Fig. 2 is shown the arrangement used in the measurement of electric absorption. S_1, S_2 and S_3 are current reversing switches which were used to reverse the current of the different parts of the bridge with relation to one another, so as to test for induction errors. In most cases no effect was found, so that it was not necessary to take a set of readings in each case with the switches in different relations. K, is a key to close the circuit through the hanging coil of the electro-dynamometer. When K was not pressed down, it closed the cross-connection of the bridge through a resistance r' equal to the resistance of the hanging coil of the electro-dynamometer. This avoided any sudden change of current and hence a change of speed of dynamo. The key K_2 was the chronograph key described above and was closed just at the time the bridge was balanced.

A few measurements were first made using the paper condensers 2 and 3. Table I shows the results. In this and the following tables R' denotes the effective value of R calculated by formula (5) and A the resistance due to the electric absorption ($A = R' - R$). T is the period of the current. The results with these condensers were very unsatisfactory, as the heating was so great that it was difficult to make accurate determinations.

The wax condenser made by Willyoung was next used.

TABLE I.

r = 6535 N = 133

Condenser	Ohmic Value R_1	R_2	R_3	R_4	Calculated R_1'	Value of electric absorption in Ohms. $\frac{A}{}$
3	34.04	2022	4600	99.7	43.86	9.82
3	34.45	2020	9090	203.4	45.22	10.77
2	34.45	2020	2475	99.7	81.46	47.01
2,3(S)	34.45	2020	8920	407.0	92.23	57.78
2,3(P)	34.45	2020	2205	51.7	47.37	12.92
2,3(P)	34.48	1991	4130	99.7	48.09	13.61
2,3(S)	34.19	1992	2031	99.6	97.83	63.64
3	34.45	1992	3325	99.6	59.72	25.27
2	34.49	1992	2414	99.6	82.28	47.79

This condenser had been made in a vacuum under pressure, and showed quite small heating by the current. By taking a series of measurements during an afternoon, the results were not affected to any great extent by the changes in temperature of the room, as these were comparatively slow. Of course there was still some slight heating by the current; but this in extreme cases did not amount to enough to occasion an error of more than one or two per cent.

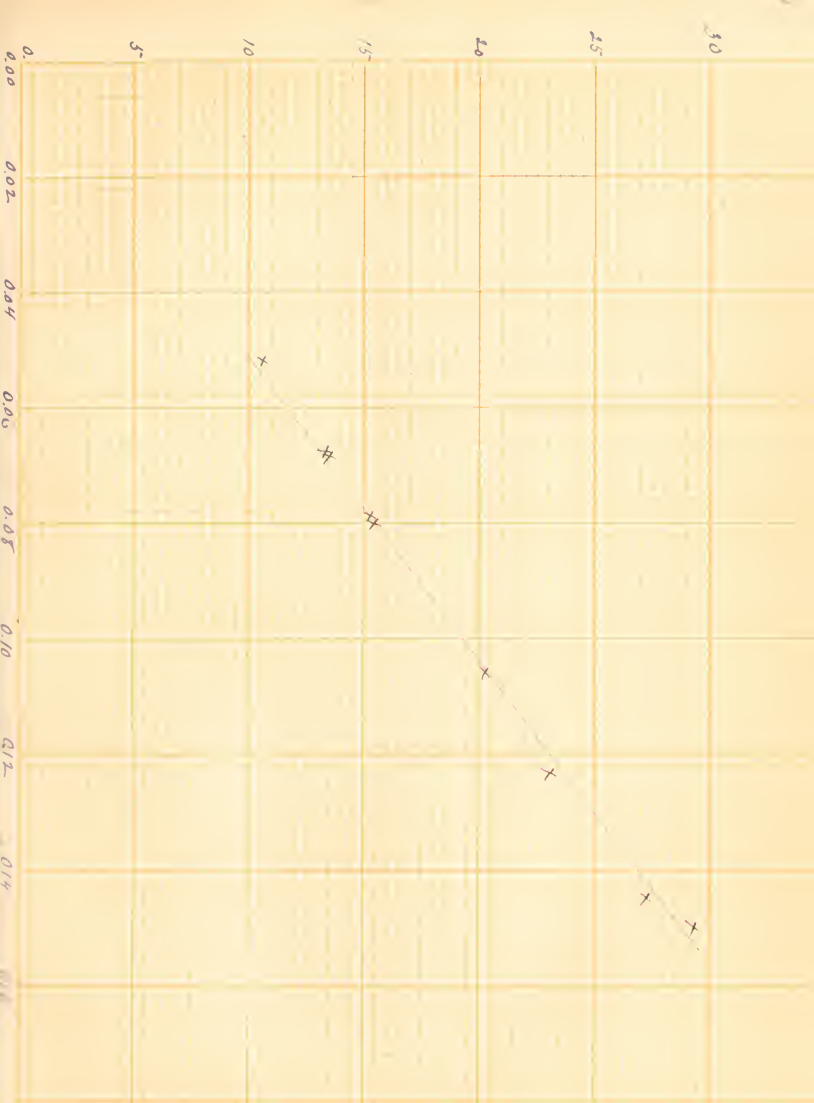
Tables II and III give the results for the absorption of all eight sections of this condenser in parallel, for different periods of the current. In Figs. 3 and 4 the above results are plotted, the ordinates representing A the resistance which in series with the condenser, would be equivalent to the elective absorption; and the abscissae represent the period of the current in seconds. The curve proves to be a straight line or A/T is a constant within the limits of error of the experiment. On account of this very simple relation connecting A and T this condenser was exceedingly convenient for a test of the method.

The first test applied was to change the relations of the resistances in the different arms of the bridge. Tables IV and V give the results for R_2R_4 nearly constant and R_1/R_4 varied. The variation in R_1/R_4 was about 200 and A/T is practically constant. The slight increase of A/T in the last two measurements is due to the larger cur-

TABLE II

r = 6020 R_z = 509.1 R_4 = 504.8

Date, etc.	R_3	R_1	Calculated R_1'	$(R_1'-R_1)$	T	$\frac{A}{T}$
3-1-991 -24-	4460	34.69	57.75	23.06	.1234	186.9
2	4676	34.71	55.09	20.38	.1066	191.2
3	4015	34.69	64.14	29.45	.1500	196.3
4	4183	34.69	61.86	27.17	.1452	193.4
5	5154	34.82	49.99	15.17	.0795	190.7
6	5108	34.81	50.44	15.63	.0808	193.3
7	5296	35.13	48.66	13.53	.0683	197.9
8	5307	35.15	48.56	13.41	.0679	197.5
9	5617	35.42	45.95	10.53	.0518	203.3



Abscissae represent period in seconds

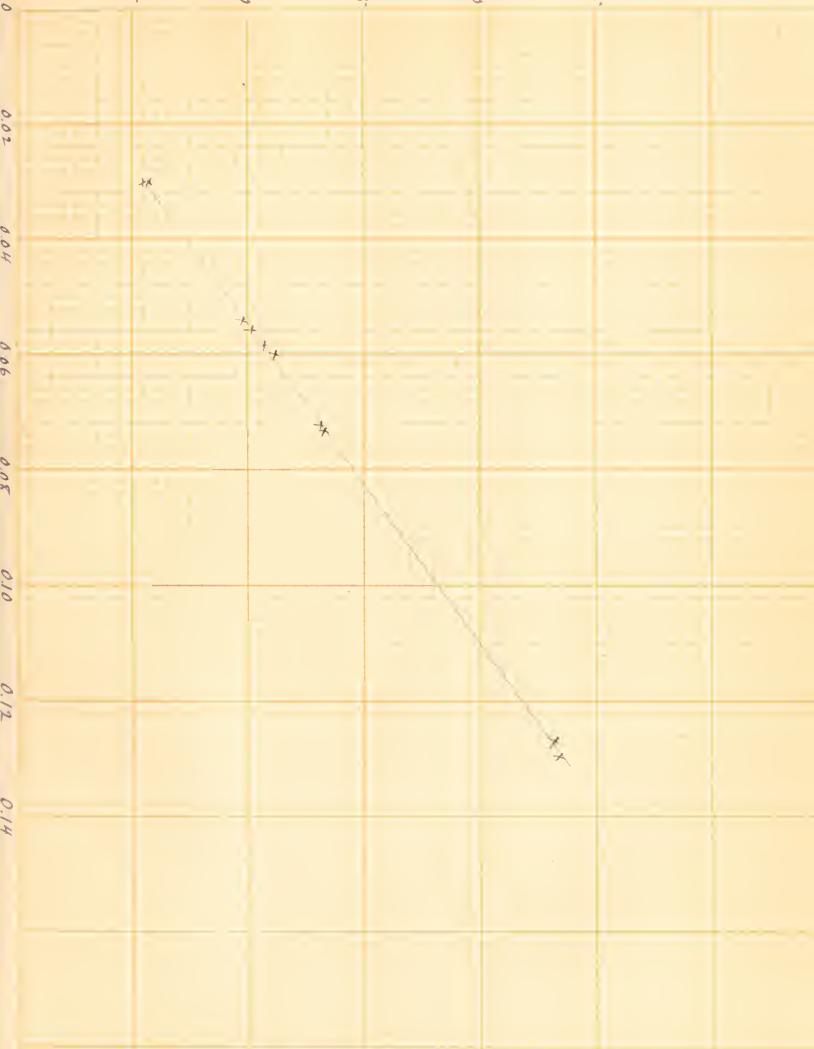
Ordinates represent A in ohms.

TABLE III.

r = 6020 R_2 = 1060 R_4 = 303.1

Date etc.		R_3	R_4	Calculated R_4'	A $(R_4'-R_4)$	T	$\frac{A}{T}$
	1	7102	34.69	44.81	10.13	.0557	181.7
2-23-09 -22-	2	7251	34.70	44.45	9.75	.0544	179.2
	3	5564	34.67	57.91	23.24	.1269	183.2
	4	5556	34.66	58.00	23.34	.1291	180.8
	5	6694	34.73	48.15	13.42	.0735	182.6
	6	6718	34.77	47.97	13.20	.0717	183.6
	7	7016	34.85	45.93	11.08	.0602	184.1
	8	7044	34.88	45.75	10.87	.0592	183.6
	9	7921	35.11	40.68	5.571	.0301	185.2
	10	7910	35.10	40.74	5.64	.0301	187.4

Figure 4.



Abscissae represent period in seconds.

Ordinates represent A in ohms.

TABLE IV.

r = 6536

Date etc.		R_2	R_3	R_4	R_1	Calculated R_1'	A $(R_1'-R_1)$	T	$\bar{T}$
3-16-09									
29	1	509.2	5561	507.2	34.61	46.57	11.96	.0640	186.7
	2	"	5581	"	34.66	46.40	11.74	.0628	187.0
	3	2071	4598	99.56	34.65	44.97	10.32	.0558	184.9
	4	"	4622	"	34.65	44.74	10.09	.0541	186.5
	5	1009.5	6750	302.9	34.71	45.42	10.71	.0572	187.2
	6	"	6785	"	34.68	45.19	10.51	.0563	186.7
	7	302.1	6732	1009.2	34.83	45.41	10.58	.0563	187.5
	8	"	6728	"	34.84	45.38	10.54	.0563	186.8
	9	99.46	4511	2044.5	35.00	45.19	10.19	.0542	188.0
	10	"	4511	"	35.04	45.19	10.16	.0536	189.7

TABLE V.

r = 6020

Date etc.		R_2	R_3	R_4	R_5	Calculated. R'_4	A $R'_4 - R_4$	T	$\frac{A}{T}$
3-3-99	1								
26	2	508.4	5854	504.3	35.53	44.10	8.57		206.7
	3	"	5859	"	35.50	43.89	8.39		203.7
	4	1009.	7121	302.7	35.34	43.01	7.67		202.9
	5	"	7145	"	35.37	42.87	7.50		199.9
	6	2068.	4905	99.49	35.36	42.09	6.73		196.6
	7	"	4922	"	35.13	41.95	6.82		195.4

TABLE VI.

R_4 = 1505.8 r = 4810

Date etc.		R_2	R_3	R_1	Calculated R'	T	$\frac{A}{T}(R_1 - R_3)$	$\frac{A}{T}$	
4-19 118	1	2070	76,600	34.68	40.70	.0312	6.02	192.9	194.8
	5	"	74,200	34.80	42.02	.0367	7.22	196.7	
	2	1009	36,120	34.76	42.07	.0375	7.31	194.9	196.0
	4	"	35,970	34.80	42.25	.0378	7.45	197.1	
	3	99.48	3,542	34.75	42.30	.0386	7.55	195.6	195.6

rent in arm 1 and a consequent heating of the condenser, and also to the fact that after the coil of the electro-dynamometer had been heated slightly, it would be cooled a small amount before its resistance could be measured. Table VI shows the results when R_4 is kept constant and R_2 and R_3 are varied. These also show A/T constant. It appears then from the above facts that A is a quantity independent of the relative values of the resistances in the different branches of the bridges.

In the next test the period was kept constant and the electromotive force acting on the condenser was varied about 300 . The results of this test are given in Table VII. The values for the higher electromotive forces are slightly greater, owing to the two heating effects mentioned above. Aside from this $\frac{A}{T}$ is constant, i.e. A is independent of the current flowing through the condenser.

The variation in A/T due to change of temperature was obtained as follows:- In an opening made in the side of the box containing the condenser, a mercury thermometer was placed. The temperature indicated by the thermometer was of course not that of the inside of the condenser but that of the outer edge. The condenser, however, was kept within a degree or two at least of the desired temperature for some six or more hours before being used.

This method gave sufficiently accurate results, as

TABLE VII.

$R_2 = 1008.9$ $R_4 = 303.6$

Date etc.		R_3	r	R_1	Calculated. R_1'	A $R_1' - R_1$	T	$\frac{A}{T}$	Volts e.m.f. acting on condenser V
3-10-99									
27	1	6649	24,410	35.45	46.14	10.69	.0523	204.3	240
	2	6593	"	35.34	46.53	11.19	.0562	199.3	"
	3	6797	12,490	35.13	45.17	10.04	.0508	197.6	152
	4	6777	"	35.15	45.31	10.16	.0511	198.9	"
	5	6606	6,348	35.03	46.56	11.53	.0584	197.3	77
	6	6473	"	34.96	47.51	12.55	.06418	195.5	"
3-14									
98	6	6591	4,807	34.73	46.60	11.87	.0590	201.2	76
	7	6803	6,371	34.79	45.14	10.35	.0515	201.1	142
	8	6732	"	34.78	45.61	10.83	.0540	200.6	"
	9	6555	18,830	35.01	46.74	11.73	.0567	203.5	243

TABLE VIII.

r = 4810

Date etc.		Temp. Cent. T	R_2	R_3	R_4	R_1	Calculated R_1'	A R_1-R_1'	T	$\frac{A}{T}$
11-10-99		19.3	709.0	2557	203.62	35.73	56.48	20.75	.0920	226.7
102	1	22.0	707.8	2195	202.6	34.90	65.35	30.45	.1167	258.2
11-2-99	7	22.2	707.8	2614	202.6	34.97	54.88	19.91	.0810	242.7
	1	16.5	709.9	3271	203.0	34.40	44.08	9.68	.0543	178.6
11-8	2	16.7	709.6	2862	203.0	34.44	50.04	15.60	.0816	191.3
106		20.2	2985	2985	203.3	34.93	48.27	13.34	.0569	234.5
	1	4.5	302.4	2696	407.3	34.55	45.87	11.32	.0752	150.9

Abscissae represent temperature at 0° C.

Ordinates represent A/T

there was no occasion for an accurate determination of the temperature. Table VIII gives the results of this investigation, and in Fig. 5 they are plotted with temperatures as abscissae and A/T as ordinates. It appears that at ordinary laboratory temperatures a variation of .1°C will cause a change of about 1‰ in the value of A/T and consequently in A for a given T. From this it appears that the slight variation noted above in the constancy of A/T would be easily accounted for by the changes of temperature due to the current or the gradual changes due to changes in the temperature of the room, since a series of observations usually occupied three hours or longer.

Table IX gives the results for the two 1/3 microfarad condensers described above connected in parallel; and in Fig. 6 the results are plotted as before. It was necessary to use the two in parallel in order to get sufficient current through the fixed coils of the electro-dynamometer. This was especially true for long periods as then the impedance of the condenser increased and at the same time the available electromotive force from the dynamo decreased.

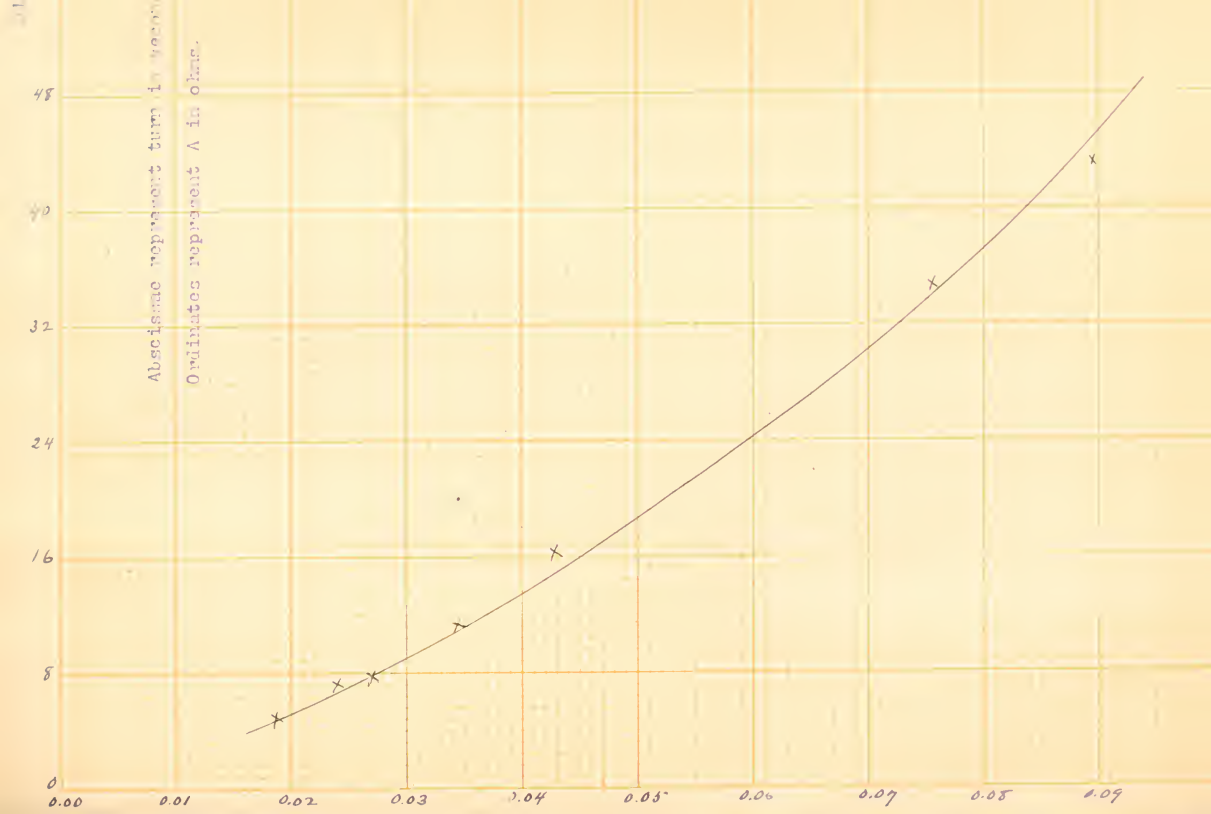

Abscisses représentent tours de roue
Ordinates représent A en ohms.
48
40
32
24
16
8
0
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09

TABLE IX

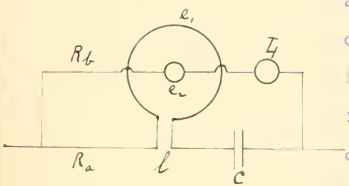
1/3 Elliot and 1/3 Troy.

R_y = 303.3 r = 4811

Date etc.		R_2	R_3	T_1	Calculated R_1'	$(R_1' - R)$	T
11-29-99	1	409.7	2984	34.07	41.66	7.59	.0271
105	2	"	2723	"	45.59	11 52	.0345
	3	"	1807	"	68.79	34.72	.0752
	4	"	1604	"	77.49	43.42	.0893
	5	"	2454	"	50.66	16 59	.0428
	6	"	3010	"	41.30	7.23	.0239
	7	410.6	3157	34.04	39.38	5.34	.0187

Capacity of a Condenser which shows Electric Absorption:- From the theory of electric absorption, as based on the heterogeneous nature of the dielectric it appears that there should be a variable value of the capacity of such a condenser depending upon the period of the current flowing through the condenser. The method chosen for the measurement of capacity was one described in the American Journal of Science, Dec., 1897.

Method:- This consisted in the use of a divided circuit. One branch, a, contained a resistance R_a, either the fixed or hanging coils of the electrodynamometer, and a condenser whose capacity c was to be studied. The other arm, b, contained a resistance R_b, a coil with which the capacity c is compared,



and either the hanging or fixed coils of the electrodynamometer. Let L be the coefficient of self-induction of the coil plus that of the coils of the electrodynamometer in that arm, and let l_1 be the self induction of the coils of the electrodynamometer in the arm b. Let an electromotive force

$$\mathcal{E} = \mathcal{E}_0 C^{i \omega t}$$

be applied to the terminals c d. Now representing the maximum values of the current by c_a and c_b and the phases by φ_a and φ_b

We have for the branch a

$$c_a \epsilon^{i(bt + \varphi_a)} = \epsilon_0 (R_a + i\, b\, l - \frac{i}{b\, c})$$

and for the branch b

$$c_b \epsilon^{i(bt + \varphi_b)} = \epsilon_0 (R_b + i\, b\, L)$$

If we adjust the resistances until there is no deflection of the electrodynamometer, we shall have the difference of phase $(\varphi_a - \varphi_b) = 90°$ or $\cos(\varphi_a - \varphi_b) = 0$. Further,

$$\frac{c_b}{c_a} \epsilon^{i(\varphi_b - \varphi_a)} = \frac{R_b + i\, b\, L}{R_a + i\, b\, l - \frac{i}{b\, c}} \qquad (12)$$

Hence since $\cos(\varphi_a - \varphi_b) = 0$ we must have the real part of this equal to zero

or,

$$(1 - b^2 c\, l)(b^2 c\, L) = b^4 c^2 - R_a R_b$$

or

$$\frac{L}{c} = R_a R_b \left(\frac{1}{1 - b^2 c\, l} \right) \qquad (13)$$

As in the case considered above a coil does not act as a self-induction alone but as a self induction in parallel with a capacity, due to the electrostatic action of the turns of the coil on one another. For this reason the above formula is not exact for any actual case; but there must be instituted

above in place of $R_L + i\,l\,L$

$$R'_L + \frac{R''_L + i\,b\,L}{\frac{i}{b\,c'}\left(R''_L + i\,b\,L - \frac{i}{b\,c'}\right)} \qquad (14)$$

Substituting this in (12) we have

$$\frac{c\,b}{c_a}\,(\varphi_c - \varphi_a) = \frac{R'_L + \frac{R''_L + i\,b\,L}{\frac{i}{b\,c'}\left(R''_L + i\,b\,\ell - \frac{i}{b\,c}\right)}}{R_a + i\,b\,\ell - \frac{i}{b\,c}}$$

$$= \frac{R'_L(1-b^2c'L)bc + R''_L\,bc + i\left\{b^2c\,c'\,R'_L R''_L + b^2c\,L\right\}}{R_a(1-b^2c'L)bc + (1-b^2c\,\ell)b\,c'R''_L - i\left\{(1-b^2c\,L)(1-b^2c\,\ell) - b^2c\,c'\,R_a\,R''_L\right\}}$$

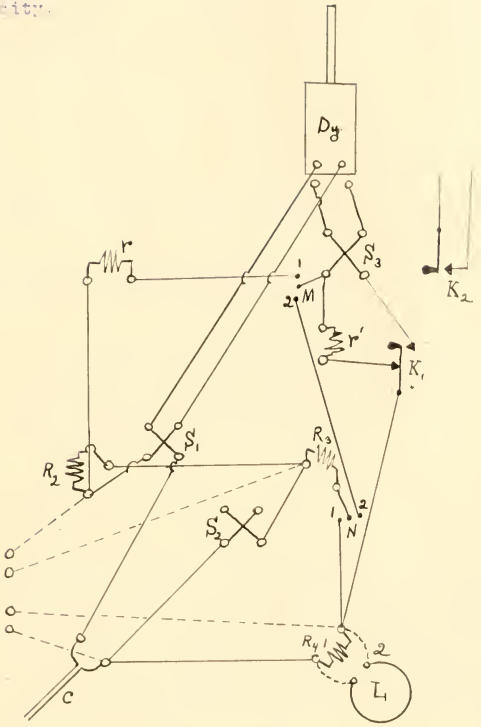
As before, the condition for no deflection is that the real part of this equals zero. Hence

$$b^2c\left[R'_L(1-b^2c'L)+R''_L\right]\left[R_a(1-b^2c'L)c + R''_L(1-b^2c\,\ell)\,c'\right]$$
$$= b^2c\left[R'_L R''_L\,c' + L\right]\left[(1-b^2c'L)(1-b^2c\,\ell) - b^2c\,c'R''_L\,R_a\right]$$

Expanding,

$$R'_L R_a(1-b^2c'L)^2c + R''_L(1-b^2c\,\ell)(1-b^2c'L)\,R'_L c'$$
$$+R''_L R_a(1-b^2c'L)c + R''^2_L(1-b^2c\,\ell)c' =$$
$$R'_L R''_L c'(1-b^2c'L) - (1-b^2c\,\ell) + L(1-b^2c'L)(1-b^2c\,\ell)$$
$$-b^2c\,c'R''_L R_a(L + R'_L R''_L c')$$

Since c' is small we can drop the term in c'^2. We have on dividing by $(1-b^2c'L)(1-b^2c\,\ell)\,c$ and rearranging the terms,



Picture 7

$$\frac{L}{c} = \frac{R'_b R_a (1-b^2 c' L)}{1 - b^2 c \ell} + \frac{R''_b R_a}{(1-b^2 c \ell)} + R''_b \frac{\ell}{c} \frac{1}{(1 - b^2 c' L)}$$

$$+ \frac{b^2 c' R''_b R_a L}{(1-b^2 c \ell)(1-b^2 c' L)}$$

or

$$\frac{L}{c} = \frac{(R'_b + R''_b) R_a}{(1-b^2 c \ell)} + R''_b \frac{c'}{c} \frac{1}{(1-b^2 c' L)}$$

$$- \frac{b^2 c' L R_a R'_b}{(1-b^2 c \ell)} + \frac{b^2 c' L R_a R''_b}{(1-b^2 c \ell)(1-b^2 c' L)}$$

Now, since in any case the last three terms are small, and $(1-b^2 c \ell)$ and $(1-b^2 c' L)$ are nearly one, they may be dropped from the last terms and we have

$$\frac{L}{c} = \frac{(R'_b + R''_b) R_a}{(1-b^2 c \ell)} + \frac{c'}{c} R''_b + (R''_b - R'_b) b^2 c' L \qquad (15)$$

Investigation - In Fig. 7 is shown a diagram of the apparatus. The arrangement is the same as in Fig. 2 except that two switches M and M' are introduced. In position 1 and with a resistance in arm 4, electric absorption can be found. In position 2 and with a coil L in arm 4 in place of the resistance, the capacity of the condenser can be compared with the standard coil L. In formula (13) R_a includes not only the ohmic resistance of branch a, but also the added resistance due to electric absorption.

TABLE X.

R_z= 1010 R_y= 912.5 r=4810 Coil A = 8.3 henry

Date		R_y	R_z	By formula(13) R_z'	T	$D=R_z'-R_z$	
4-20	1	4080	224.6	225.8	.0108	1.2	
	2	4092	224.8	225.2	.0488	.4	
	3	4074	225.0	226.2	.0182	1.2	
	4	4034	225.0	228.4	.0182	3.4	with .01 Microfarad in parallel with coil.

If Formula (15) is used in calculating R_z' it is 226.2. Hence D = 1.2 as without condenser.

A preliminary investigation was carried out to find whether the correction due to the capacity of the coil c' were appreciable and if so, to ascertain its amount.

The method chosen was as follows: An arrangement was made as in Fig. 2 except that in place of the condenser c the coil L with which the capacity of the condenser is to be compared is placed in arm 1 of the bridge. The arrangement was first balanced with a direct current; and the values of R_1 as measured and as calculated from R_2, R_3, R_4 were the same. In table X are given the results for R_1' as calculated for three periods of the current. In all three cases R_1' is greater than R_1 by an amount D. A third observation was made with a condenser of .01 microfarad capacity shunted across the terminals of the coil. The corrected formula (7) was tested in this way. The result gives the same value of R_1 with or without the condenser, thus verifying the formula. By assuming the quantity D as entirely due to the electrostatic action of the coil, which if not absolutely true, the formula will at least give a value of c' the equivalent capacity of the coil, which may be used as the limiting value. This value of c' is .006 microfarads. In formula (15) the last term will be very small as compared to the first for any values of b used in this work. The second term will in the most unfavorable circumstances amount to only 1 part in 10,000 so that the correction due to the electrostatic action

TABLE XI

r = 4811 R_2 = 707.8 R_4 = 202.6 L=5.302 in 2 and 5.318 in rest.

Date etc.		R_3	R_1	Calculated R_1'	A $(R_1'-R_1)$	T	$\frac{\Lambda}{T}$	Mean $\frac{\Lambda}{T}$
3-16	1	2195.	34.90	65.35	30.45	.117	261.	253
	7	2614	34.97	54.88	19.91	.816	244	

		R_a	R_b	T	A	Corrected for electric absorption R_a	Corrected for electric absorption R_b	$10^6 \times \frac{L}{S_a S_b} = c$	
	2	238.2	2604	.121	30 6	268.8	2634	7.574	M.F.
	3	228.8	3053	.103	26 1		3079	7 549	
	4	328.2	2119	.103	26.1		2145	7.554	
	5	431.5	1602	.102	25 8		1628	7.569	
	6	531 5	1302	0908	23.0		1325	7 552	

of the coil may be entirely neglected.

The next point investigated was the correction due to the electric absorption. In these observations the absorption was determined, the capacity then measured, and the electric absorption again determined. In Table XI are given the results with the condenser in one arm and then changing to the other, and also changing the resistance in series with the condenser. The results are corrected for electric absorption and the change in L due to the change of c and the coil from one arm to the other, caused by the coils of the electro-dynamometer having different coefficients of induction. The greatest difference between two determinations under these different conditions is 3 parts in 1,000.

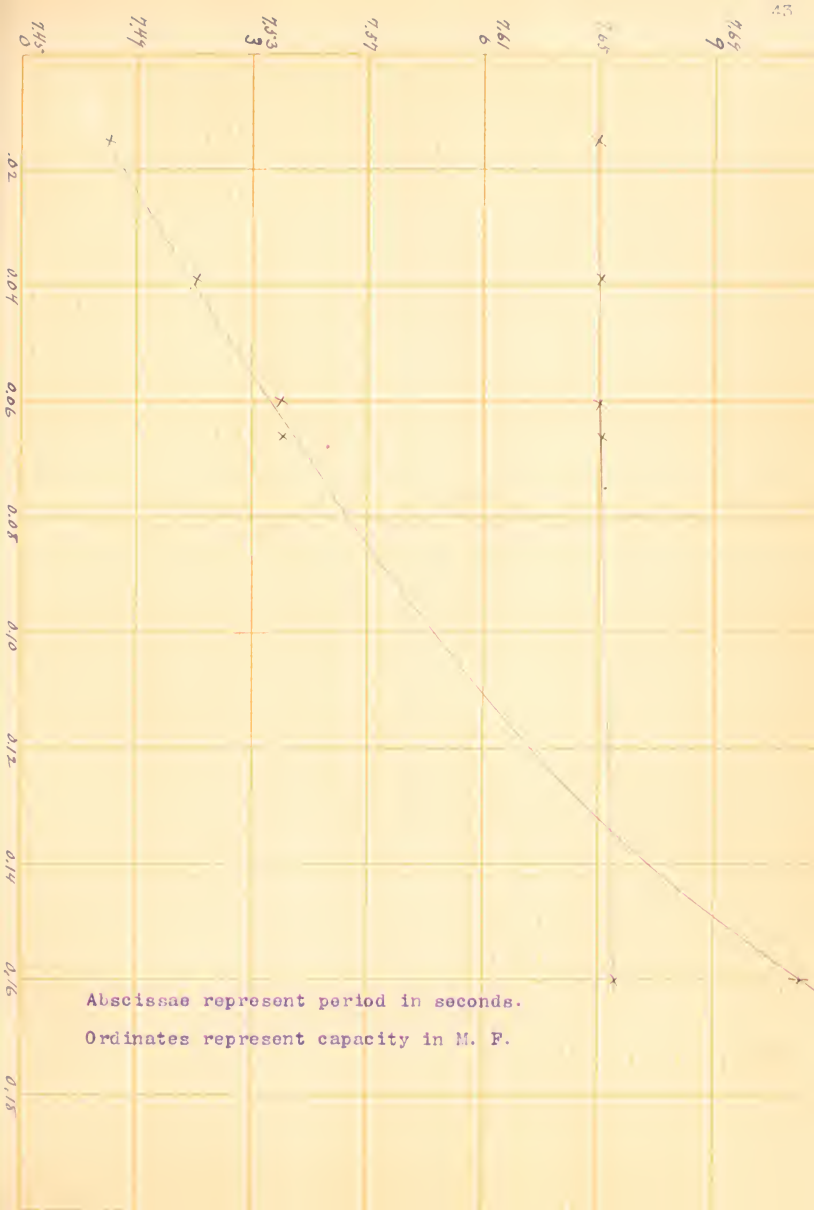
The change of capacity with the period of the current was now tried. Table XII shows the results of the investigation. The error of each observation has a limit of about 1 part in 1,000, if the observations are compared among themselves while the actual errors, as compared with the true ratio $\frac{b}{c}$ may be in error two or three times this limit; but we are not particularly concerned here with the actual value, but merely the change with change of period. In Fig. 8 the results are plotted on two scales. The results for the capacity seem to agree very well with the theory. The

TABLE XII

Date L = 5.316

R_2	R_3	R_4	R_1	r	R'_1	A	T	$\frac{A}{T}$
709.0	2556.8	203.62	35.73	4811	56.46	20.73	.0919	225.8

				Corrected for electric absorption.			
R_4	R_5	T	A	R_a	R_b	$10^6 \times \frac{L}{R_a R_1} = c_1$	$c = \frac{c_1}{1+Dc_1}$
429.2	1589.1	.160	36.2	429.2	1605.3	7.719	7.719
429.3	1628.1	.0654	14.8	429.3	1642.9	7.540	7.540
429.3	1640.0	.0389	8.8	429.3	1648.8	7.513	7.512
429.4	1651.0	.0151	3.4	429.4	1654.4	7.487	7.480
429.4	1623.0	.0601	13.8	429.4	1641.8	7.543	7.543



Abscissae represent period in seconds.

Ordinates represent capacity in M. F.

condenser has a capacity which shows a slight variation with the period of the current. The capacity increases with increase in the period of the current, as shown by the theory, and in amount it is somewhat less than as the square of the period

DETECTION OF SHORT CIRCUITS.

A very useful application of the Wheatstone bridge is its use for the detection of short circuits in coils of wire. If a mass of metal or a closed coil of wire is held near a coil of wire carrying an alternating current, there will be induced in it certain currents. Hence there must be more energy supplied to the primary circuit to keep up the current. This extra consumption of energy will manifest itself by an apparent increase in the resistance and consequently a greater i^2R loss. This fact is made use of in the following method for the detection of short circuits in coils and was suggested by Professor Rowland.[1)]

The method is as follows:- The connections are as in Fig. 2, with the exception that a coil of wire is introduced in arm 1 in place of the condenser. If the resistances are now balanced until there is no deflection of the electrodynamometer, and a mass of metal approaches the coil, there will be a deflection of the electrodynamometer, owing to the increase in the effective resistance of the arm 1. If a coil of wire whose ends are not connected be laid on top of the coil in arm 4 there will be no deflection, while if the ends are connected or if there is a short-circuit in the coil there will be a deflection.

Table XIII shows the sensitiveness of the method. The

1) Am. Journal of Science. Dec. 1887.

coil A of 4.3 henrys was used. The bridge was balanced and then small coils of wire the same size as the inner diameter of the coil were placed on the coil and the deflection noted; and the apparent increase of resistance was determined in the same manner as the electric absorption in the case when the coil 1 was replaced by a condenser. Column 1 gives the deflection after the coils were placed on the large coil in arm 1 and 2 is the apparent increase in resistance of arm 1. It appears from this table that with a coil of the size used a short circuit in another coil of the same size could be detected, even though the coil were of quite fine wire and only one turn was crossed. Other conditions being the same, the sensitiveness varies directly as the cross section of the wire in the coil to be tested, if the resistance of the contact between the two ends of wire is neglected. In cases where small coils are to be tested, the sensitiveness may be increased by filling the centre of the coil with iron. And of course as short a period of current as available should be used.

TABLE XIII.

R_2= 1010 R_y= 1507 R_1= 225.7 T = .015

No. of turns	Diam. of wire	R_3	Uncorrected R_1'	Δ	δ_c
0		6711	226.8	0	
2	#14	1.59	5882	250.8	9.15cm	32.0
3	#22	.62	6497	234.3	2.21	7.5
5	#25	.44	6529	233.4	1.90	6.6

Hysteresis.

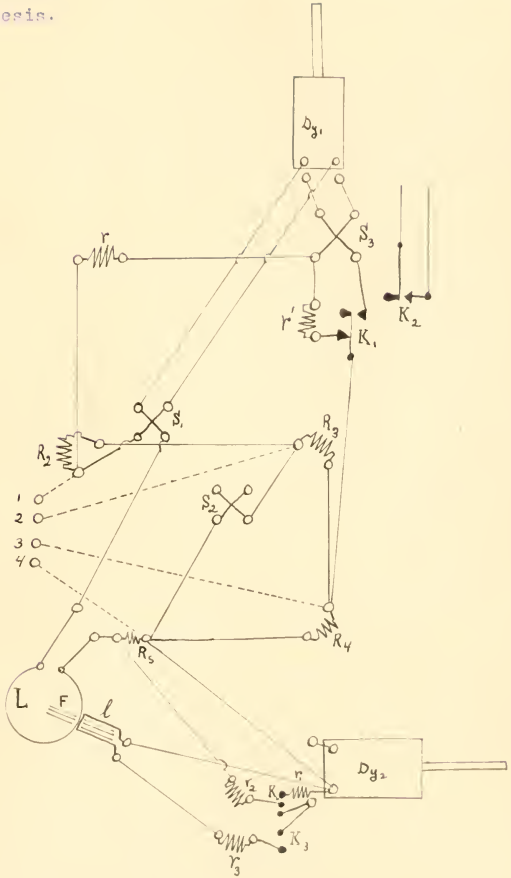


Figure 9

The arrangement used for the determination of hysteresis and Foucault current losses is the same as for the determination of electric absorption, with several additional elements.

Fig. 9 shows the arrangement used. All the arms are the same as in Fig 2 except arm 1. Arm 1 contains a coil L. In this coil is placed the iron to be tested, F. l is a small coil of wire surrounding the iron and used to determine the induction through the iron. The two coils of the electrodynamometer D_2 are connected in series. The key K_3 serves to put the coil l, the resistance r_3 and the coils of the electro-dynamometer in series. By noting the deflection produced by the current induced in l, the induction may be calculated, the electro-dynamometer having been previously calibrated. R_5 is a small resistance in arm 1. A small current is shunted off from the terminals of R_5 and can be sent through the resistance r_1 and the electrodynamometer by the key K_4; and thus the total current in arm 1 may be determined.

Measurement of the Current:- The calibration of the electrodynamometer showed the current to be quite accurately proportional to the square root of the deflection ($\sqrt{D}$). A deflection of 1 cm corresponds to a current of .00213 am-

pere. We shall then have the total current in ord l

$$c_{/} = .00913\sqrt{5}\ \frac{P_2 + P_3}{P_5}$$

In place of measuring the current by an electrodynamometer, in some cases a small ammeter might be used, but as the current enters as a square in the energy loss, it must be accurately determined.

Determination of the Induction:- For the determination of the induction several different forms of the secondary coil were tried. Coils of the same size as the internal diameter of the coils A and C were first used. These were found unsuitable, however, as a considerable current was induced in them, even when they contained no iron, and their coefficient of self induction was not negligible. The best form was found to be a coil of 50 turns just large enough to contain the iron used. No measurable current was induced in this, when it contained no iron, and its self induction was negligible. The E. M. F. around the circuit when $K_{/}$ is closed will be $\mathcal{E}$, where

$$\mathcal{E} = \sqrt{5} \times .00913 \times P \text{ volts}$$

Now let N be the number of turns in the coil A and S the average cross-section of the iron surrounding the coil L, and we have the induction per sq. cm. B

$$B = \frac{\sqrt{E} \times .00213 \times R}{T \times 4.44 \times n} \times 10^{8} \text{ c.g.s. units}$$

where n is the number of complete periods of the current.

Energy loss:- The energy loss due to hysteresis is ordinarily expressed as a certain loss per c.c. of iron per cycle. Energy loss = $i^2 R t$.

If c_1 is the current in amp. 1, R the total added resistance due to the hysteresis and Foucault currents, v the volume of the iron used, we have the energy loss per cycle

$$c_1^2 \frac{L}{v} T$$

since t will equal T, the period of the current.

Experiment:- The iron used was ordinary transformer iron. The plates were L shape, and could be fitted about the coil C very nicely. In the first place the uniformity of B was tested for different quantities of iron. A slot was cut in the centre of one side of an L plate, thus dividing the side into two parts, which were made as nearly equal as possible. A coil of wire of one hundred turns was wound on each of these parts of the iron. By noting the deflection produced by the two coils in succession when the iron was placed on the coil A, the relative value of B close to the coil A and farther away could be tested. By placing on the coil different numbers of the plates and placing the test plate at different

TABLE XIV.

No. of pieces of iron	Test plate on outside of bundle			Test plate next to out side plate		Two plates outside of test plate		
	Lower coil	Upper coil	var B	Lower coil	Upper coil	Lower coil	Upper coil	
2	6.51	6.03	3					
4	4.14	3.82	3					
6	3.60	3.48	1.9	3.60	3.62			
8	2.65	2.55	2	2.71	2.71			
10						2.75	2.65	2

points the uniformity of B was tested.

Table XIV shows the results of this test. It appears from the table that with either 8 or 10 pieces of iron B is practically uniform. With fewer pieces the induction was greatest next the coil, and when more were used the magnetism was not as great in the central pieces.

Table XV shows the results of a series of measurements, and in Fig. 10 a curve is plotted showing the relation between B and the energy loss per cycle.

TABLE XV

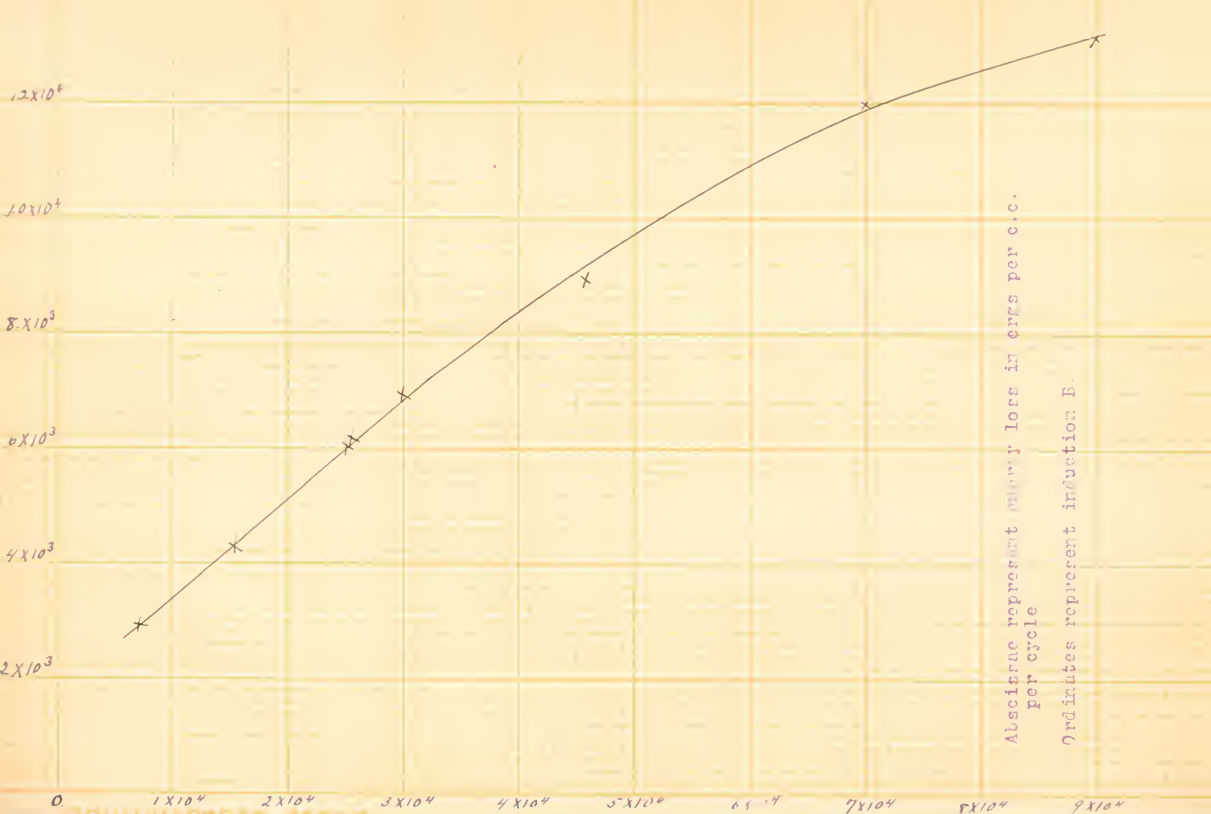
Weight of iron = 340 g. Volume = 43.3 cc. Area section=.981 sq. cm.

D_1 = deflection on dynamometer by current c, D_2 same for secondary current.

Date 3-21-00

	B_2	B_3	B_4	B_1	D_1'	H	T	D_1	D_2	r_1	r_2
1	913.2	2945	1011	148.3	313.5	165.2	.0277	3.62	3.18	166.7	59.72
2	505.9	2140	1508	"	356.5	207.8	.0280	5.85	7.42	"	"
3	208.6	1106	2016	"	380.2	231.9	.0275	8.78	14.70	"	"
5	1769	2408	507.5	"	372.8	224.5	.0276	9.00	3.00	"	129.72
6	1563	2100	"	"	377.7	229.4	.0273	10.61	3.86	"	"
8	1060	1431	"	153.0	375.9	222.9	.0267	4.00	6.95	377.0	"
9	503.1	1671	1010.5	153.0	304.2	151.2	.0254	9.60	14.10	"	"
11	206.9	1639	2014.	153.1	254.1	101.1	.0255	18.10	14.36	"	143.8

	B	c,	H per cc.	C^2H,T
1	2.88×10^3	.0262	3.815	7.10×10^3
2	4.32×10^3	.0337	4.799	1.524×10^4
3	6.17×10^3	.0412	5.356	2.51×10^4
5	6.07×10^3	.0418	5.185	2.50×10^4
6	6.86×10^3	.0454	5.298	2.88×10^4
8	8.92×10^3	.0576	5.148	4.56×10^4
9	1.207×10^4	.0891	3.482	7.00×10^4
11	1.318×10^4	.1226	2.335	8.93×10^4


Abscisse represent energy loss in ergs per c.c. per cycle
Ordinates represent induction B.
.12X10⁴
.10X10⁴
8.X10³
6X10³
4X10³
2X10³
0
1X10⁴
2X10⁴
3X10⁴
4X10⁴
5X10⁴
7X10⁴
8X10⁴
9X10⁴

SUMMARY.

From the above it appears that the method described is a perfectly good method for the measurement of electric absorption. In all cases tried the electric absorption has acted as a resistance in series with a capacity This resistance is independent of the current. The temperature has a decided effect. The value of this absorption increases very rapidly with rising temperature. The theory as given above appears to be verified by the results in as far as a condenser possessing electric absorption may be considered as a capacity in series with a resistance, both of which depend upon the period of the current. While the variation is in the proper direction in each case, its rate of change with the period does not agree with the theoretical formula, especially in the case of the wax and paper condenser.

It also appears that the method given is a good one for the determination of the capacity of a condenser which shows electric absorption. If electric absorption is corrected for, the capacity of such a condenser is a quantity which can readily be determined, and may be compared with a self induction standard to 1 part in 1,000. It also appears that no correction is necessary for the electrostatic action of the turns of the standard coil on one another, at least if the relative size of coil and condenser is properly chosen.

and if the resistance of the coil is not too large as compared with the total resistance in the branches of the circuit

The method given for the detection of short circuits in coils proves to be an exceedingly sensitive one.

The method described for the measurement of hysteresis and Foucault current losses gives very good results. Its chief advantage lies in the fact that a very small quantity of the material is necessary. In the experiment only 360 grams were used. Much less than this amount could be used with nearly as great accuracy as in the case given.

For the value of the resistance H could be increased by decreasing the period of the current used.

In conclusion I wish to express my sincere appreciation of much assistance received from Professors Rowland and Ames during three years spent in study at the Johns Hopkins University. The investigation was suggested by Professor Rowland, and the methods used were those devised by him and described in the articles noted above.

Johns Hopkins University,
May 1, 1900.

Louis Maxwell Potts.

BIOGRAPHY.

Louis Maxwell Potts was born at Canonsburg, Pa., on October 30, 1876. His early education was received in the public schools of Canonsburg. In 1891 he graduated from the Canonsburg High School, and during the following winter (1891-92) attended the Jefferson Academy, Canonsburg, Pa. In 1892 he entered Washington and Jefferson College, Washington, Pa., and received the degree of Bachelor of Arts in 1896. He taught in the Washington High School, 1896-97; and since that time has attended the Johns Hopkins University as a graduate student in Physics. During the three years spent in study there he has pursued the following courses in mathematics: solid analytic geometry and theory of equations under Professor Hulburt; theory of functions and differential equations under Dr. Cohen; in Chemistry, lectures and laboratory work in Physical Chemistry under Dr. Jones, and an experimental course in inorganic preparations under Dr. Penouf; in Physics, the major undergraduate course, lectures on physical optics, hydrodynamics, chemical dynamics, spectrum analysis and theoretical mechanics, under Professor Ames and lectures on electricity and magnetism, heat conduction and physical optics under Professor Rowland. He also attended and took part in the Journal meetings conducted by Professor Rowland, and the Physical Seminary conducted by Professor Ames. In 1899-1900 he was student-assistant in Physics.